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ABSTRACT

An analytical framework is described through which voluntary attrition can be predicted. The approach incorporates Marshallian Utility Theory and a Maximum Likelihood Estimation procedure to evaluate a specific individual's propensity to attrit. The approach was tested twice at the United States Air Force Academy where it was able to correctly predict over a third of the voluntary losses a priori on a by-name basis. These results indicate that the approach has practical usefulness as an operational tool. (Author/RC)

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PREDICTING ATTRITION: AN EMPIRICAL STUDY AT THE UNITED STATES AIR FORCE ACADEMY

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TM005 795

PREFACE

The research described in this report was initiated by the Assistant for Personnel Plans, Programs and Analysis at the Air Force Military Personnel Center, Randolph Air Force Base, Texas. It describes an analytical framework through which motivational attrition can be predicted. The procedure was initially tested at the United States Air Force Academy to determine the feasibility of the approach. Based upon results of this initial testing it appears the prediction system described herein has some practical applications.

The Air Force Academy Superintendent and his staff have been apprised of the underlying model and have expressed interest in our results. These findings are particularly applicable for use by USAF analysts involved in first term attrition studies.

Although the conclusions are self-contained, a knowledge of the economic theory of utility and of statistical regression, logit and probit theory would be helpful in understanding the model and its estimation.

This report has been reviewed and is approved for public release.

WINFIELD S. HARPE, Colonel, USAF
Assistant for Personnel Plans, Programs
and Analysis

SUMMARY

The research contained in this report describes an analytical framework through which voluntary attrition can be predicted. The approach incorporates Marshallian Utility Theory and a Maximum Likelihood Estimation procedure to evaluate a specific individual's propensity to attrit. The approach was tested twice at the United States Air Force Academy where it was able to correctly predict over a third of the voluntary losses a priori on a by-name basis. These results indicate that the approach has practical usefulness as an operational tool.

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I. INTRODUCTION

Background

Each year some percentage of airmen and officers fail to graduate from various formal technical training programs which are designed to provide the individual with essential knowledge to become a more productive member of the United States Air Force.

For example, the Undergraduate Pilot Training (UPT) Program has historically experienced a 19.4 percent attrition rate.[1/] Each individual who "washes out" of the program represents a real loss of Air Force dollars, the magnitude of which depends upon what stage of training the elimination occurs.

The Air Force has recognized the problem for years and has directed much research into better ways of selecting candidates for its training courses. Although previous military studies have yielded valuable insights into the role certain factors play in determining an individual's likelihood of success, few have attempted to predict success or failure for a specific individual.

Research Objectives

The research described herein has two primary objectives:

(1) develop a general method to predict individuals who will attrit from various Air Force training programs.

(2) evaluate the methodology in a simulated operational context for potential usefulness.

1/ ATC Management Summary, ATC/DCS Comptroller, Management Analysis Division, 7 Aug 75, p. DO-13.

Three criteria were established to evaluate the methodology's usefulness. First, the procedure must utilize variables which can be collected before the actual selection process takes place. Second, the mathematical techniques must be general in nature to ensure it can be applied to other Air Force applications. And third, the methodology must yield a prediction for a specific individual.

Outline of Report

The remainder of this report is divided into four sections. Section II describes previous research and the Conceptual Model used in the present report. Section III describes the initial study at the United States Air Force Academy to evaluate the usefulness of the procedure in relation to potential application to other Air Force programs. Section IV discusses a follow-on empirical test of the procedure which predicted attrition from the Class of 1979 using a prediction system developed on the Class of 1977. The empirical test was designed to demonstrate that the methodology could predict attrition a priori on a by-name basis. The conclusions of the exploratory investigations at the United States Air Force Academy are included in Section V along with implications for applications to other training programs.

Several appendices have been included to provide source material for the technically oriented reader. The appendix of greatest interest is Appendix D which describes the Maximum Likelihood Estimation (MLE) procedure used in this report. Other appendices address the data base (Appendices B, C, and E) and alternative estimation procedures (Appendix A).

II. ESTIMATING THE LIKELIHOOD OF SUCCESS OR FAILURE

In this section we describe the conceptual approach and rationale for the estimation procedure used in this study. We begin with a brief discussion of previous military studies.

Previous Military Studies

In view of the magnitude of attrition costs within the Air Force, it is not surprising that several attempts have been made to identify its causal agents. Three studies represent the foundation upon which we built the analytical framework described in this report. They are described in the following paragraphs along with the many insights they provided.

In their study, "Predicting the Potential for Active Duty Success of Rehabilitated Air Force Prisoners," Smith, Gott and Bottenberg attempted to develop a prediction system which could identify which retrainees should be released from the Air Force and which retrainees should be returned to active duty. Their objective was to demonstrate that such a system could be derived from a statistical analysis of a large number of personal attributes. The data base consisted of 139 variables collected on each individual referred to the Air Force's 3320th Retraining Group. These variables were classified as either (1) pre-military background, (2) general military, (3) offense variables or (4) measurements while in the Retraining Group. Of the 6,799 individuals referred to the Retraining Group between the years 1952 and 1963 only 1,303 individuals had the necessary data recorded on them. This group represented the computational sample. A cross validation sample of 583 later cases was assembled by selecting only those retrainees which had the requisite data. Before the two multiple regression analyses were carried out, the original 139 data elements were expanded into a total of 687 variables. After the analysis was completed, they derived two regression equations. The first equation included 61 variables and the second included 13 variables. They subsequently applied the equations to the cross validation sample to determine how efficiently active duty success or failure could be predicted. They concluded that

in order to maintain an acceptable level of error they could predict only on extreme cases and this may have prevented the eventual widespread operational use of their prediction system. In any case, it certainly demonstrated that a large number of explanatory variables will not necessarily yield significantly improved predictive ability unless a meaningful relationship can be established with the criterion.

In a later study incorporating relatively few explanatory variables, Guinn[2] attempted to (1) validate a psychometric instrument called the History Opinion Inventory (HOI)[3] which exhibited a modest correlation with the criterion groups she defined and (2) assess the marginal gain in predictive ability of including additional biographical/aptitudinal data. The purpose of the study was to determine whether or not the HOI and the additional data could be used for the early identification of basic airmen who would be unable to adapt to a military environment and subsequently be discharged. Guinn utilized data gathered on approximately 15,000 basic airmen in 1972. She catagorized this data into several criterion groups. For her analysis tests of significance between the various criterion groups were accomplished and several regression analyses performed. She reported that 25 percent of the undesirable losses with less than a 6.5 percent error could be correctly classified. However, once two or more variables are used to predict an individual's probability of success some method must be found to determine (1) which factor is more important and (2) to what degree.

Because of the statistical difficulties associated with Ordinary Least Squares (OLS) Regression Analysis (involving a qualitative dependent variable) which have been revealed in academic literature, the question remains as to whether Guinn's results could have been enhanced through the use of an alternative statistical technique.

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[2] Guinn, Nancy, Allan L. Johnson and Jeffrey E. Kantor, Screening for Adaptability to Military Service, AFHRL-TR-7530, Lackland AFB, TX: Personnel Research Laboratory, Air Force Systems Command, May 1975

[3] Lachar, D., J.C. Sparks and R.N. Larsen, "Psychometric Prediction of Behavioral Criteria of Adaption for USAF Basic Trainees." Journal of Community Psychology, 1974, 2(3), pp. 268-277.

In a more recent study conducted at the Center for Naval Analyses, Lockman, Jehn and Shughart compared the OLS regression model to the Logit regression model (many observations per cell) with respect to their ability to forecast premature losses among naval enlistees. Their data consisted of biographical/aptitudinal variables collected on some 66,000 FY73 male accessions which was later matched to appropriate loss data. They grouped individuals by combinations of variables and then calculated the loss rate for each group. Of the 180 possible groups only 148 contained data. These 148 groups became the units of observation for the comparative regression analyses of categorical variables upon loss rates. They reported that the simpler linear model compared very favorably to the Logit model so they decided to adopt the former in estimating the survivability odds.

Though their analysis was adequate with respect to their stated objectives and they recognized the statistical problems[4/] associated with OLS regression their approach cannot be adopted for our purposes. Although they stated they could predict the probability of any individual attriting during the first year of service, they were actually forecasting the loss rate for a specific group with a given set of categorical attributes. Notwithstanding, no provision was made for the increased uncertainty associated with forecasting the behavior of a specific individual and a deterministic relationship was assumed between behavior and explanatory characteristics. Because of these difficulties and the absence of cross validated results a different approach was sought.

4/ The problems identified were heteroskedasticity and the Bernoulli nature of the error term. (See Appendix A)

Rationale for Using Maximum Likelihood Estimation

> "Although Maximum Likelihood methods for the analysis of qualitative data have been discussed in literature for years, econometricians and other analysts of qualitative socio-economic data continue to use inappropriate and overly restrictive methods." [5/]

Application of OLS regression to problems involving dichotomous dependent variables can yield highly misleading results since the distributional characteristics of the error term are no longer in consonnance with the classical assumption of normality. First, standard tests of significance, with respect to the estimated coefficients do not apply since the estimates are biased and inconsistent. Second, the traditional measure of performance, the multiple R^2 is no longer meaningful for comparison with non-linear estimation methods since the errors are not commensurable. [6/] And third, the estimated probabilities can vary outside the unit interval which make interpretation difficult. Notably, Nerlove concluded that "...we can always improve on the least squares estimation (whether or not it is corrected for heteroskedasticity) since it is a linear estimator." Thus the problem revolves around the misspecification of the functional form. Appendix A contains a more rigorous treatment of the statistical difficulties associated with OLS regression and an empirical comparison of predictive capability with Maximum Likelihood.

Maximum Likelihood Estimation (MLE) methods are ideally suited for analyzing relationships involving a dichotomous dependent variable. However, most MLE methods assume a deterministic relationship when predicting attrition as a function of personal attributes and make no provision for the increased uncertainty in forecasting the behavior of a specific individual. Appendix D describes a general MLE method which overcomes these deficiencies. The remainder of this section describes a behavioral paradigm which provides the conceptual underpinnings of the mathematical technique.

[5/] Nerlove, Marc and S. James Press, Univariate and Multivariate Log-Linear and Logistic Models, (Santa Monica California: Rand Corporation R-1306-MA/NIH, 1973), p. V.

[6/] Ibid., p. 7.

The Conceptual Model

We assume that an individual faced with two alternative choices (e.g. buy versus not buy) assigns a utility to each. From the individual's point of view the choice is deterministic i.e. has knowledge of all the information he uses to make a decision. From the observer's point of view there is a systematic component and a random component. The systematic component includes all information available to simulate the decision process while the random component represents omitted information. By applying utility maximization to the systematic component and developing a decision rule to state which alternative will be chosen a fraction of the cases will be predicted correctly.

For example, at the United States Air Force Academy the Maximum Likelihood technique described in Appendix D estimated a cadet's utility for attrition (voluntary) and yielded a probability of that alternative being selected. If a cadet's utility for attrition was higher than the estimated mean utility for attrition[7] of all cadets, he was predicted to leave prior to graduation. The following section describes in more detail how this procedure was tested at the United States Air Force Academy to evaluate its potential usefulness in other Air Force applications.

7/ The mean utility for attrition should be interpreted as the estimated mean point of indifference for the sample with respect to the two alternatives. (see Appendix D)

III. THE UNITED STATES AIR FORCE ACADEMY INITIAL STUDY

This section describes an initial test conducted at the United States Air Force Academy which was designed to evaluate the conceptual approach and estimation procedure used in this report for potential application to other Air Force programs. The Air Force Academy was selected to test the methodology because of the extensive data maintained on each candidate/appointee/cadet.[8/]

Background

Historically the Air Force Academy has experienced a cadet attrition rate which has ranged between 28 and 46 percent. An estimated two-thirds of those who attrit possess a significant motivational component whereby the separation action is initiated by the cadet. The remaining attrition can be roughly classified as either academic or miscellaneous. Academic attrition generally results from formal board action after the cadet has failed to meet the minimum academic standards for retention while miscellaneous separations include such reasons as hardship, medical and accidental death. Upon separation, each cadet has his record annotated with a two digit code (see Appendix E) which best describes his reason for leaving. Since the conceptual model described in Section II precludes involuntary action on the part of the cadet this initial test was designed to predict only motivational (voluntary) attrition.

Data

The data used included information from four major sources--The Air Force Academy General Information Questionaire (GIQ), the Survey of High School Activities (HSA), the Strong Vocational Interest Blank (SVIB) and other data relating prior academic achievement.

8/ A candidate is an applicant who has not yet been tendered an appointment to the Academy. An appointee has been tendered an appointment and assumes cadet status after arriving at the Academy and taking the Cadet Oath.

General Information Questionnaire (GIQ): The GIQ is a questionnaire designed to provide both personal background data and information about factors that influenced the candidate to apply to the Academy. The GIQ is mailed to the candidate for completion and is returned to the Academy prior to his arrival.

Survey of High School Activities (HSA): The purpose of the HSA is to provide information about each appointee's participation in extracurricular activities while in high school. Included are the varsity sports he participated in and the fraternal and elective organizations of which he was a member. The survey is completed by each cadet within two weeks of his arrival at the Academy.

Strong Vocational Interest Blank (SVIB): The SVIB is a 399 item self-report inventory that assesses a cadet's[9/] interest in various occupational and general interest areas. Eighty-four scales can be constructed using responses to items that have been previously identified as being related to specific occupations.

Prior Academic Achievement: A transcript of each candidate's high school academic record is transmitted to the Academy which includes course grades and class standing. In addition performance on the College Entrance Examination Boards (CEEB), Scholastic Aptitude Test (SAT) or American College Test (ACT) are sent to the Academy. These scores are weighted to develop several indices which are used in the selection process. These are: prior academic record (PAR), scientific index, and non-scientific index. Other indices are generated which incorporate additional non-academic information. These are: the athletic index, non-athletic index, leadership composite, weighted composite, and academic composite.

Test Methodology

Certain data elements were extracted from the four primary data sources which were then used to construct a record on each cadet. Each record was annotated with the cadet's status as of 1 June 1975 (Ø if still enrolled, 1 and discharge code if not enrolled). Any record which was missing one or more of the principle variables

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9/The SVIB has been administered after arrival at the Academy, but within the first two weeks of Basic Cadet Training. A revised version of the SVIB, the Strong-Campbell, is currently being administered to all candidates for the Class of 1980.

was eliminated from the sample. The principle variables are listed in Appendix B. Appendix C contains the file layout for a typical record.

The test was conducted using the Classes of 1976 and 1977. A prediction equation and critical limit (prediction system) were estimated for the Class of 1976 using the estimation procedure described in Appendix D. This prediction system was then applied to the Class of 1977 for cross validation. Table 1 shows the sample sizes for the two classes.

TABLE 1

SAMPLE SIZES FOR INITIAL TEST

	1976	1977
Cadets Still Enrolled	916	937
Motivational Attritions	237	246
TOTAL IN SAMPLE	1153	1183

Results

The Maximum Likelihood Estimation (MLE) procedure correctly classified 32.1 percent of the actual attritions and 94.2 percent of the actual successes (Table 2). Figure 1 shows that over 59 percent of the predicted attrition group did, in fact, leave the Academy within their first two years while only 15.8 percent of the predicted success group separated. All of these separations were classified by the Academy as possessing a significant motivational component.

The value of incorporating such a procedure into the selection process, assuming the validity of the SVIB in the pre-selection environment, is that the Academy could have conceivably eliminated from consideration those candidates predicted to attrit. However, we strongly recommend that any eventual use of the procedure be only a compliment to and not a substitute for the existing selection process. In this way the opportunity wrongly denied to an individual can be minimized.

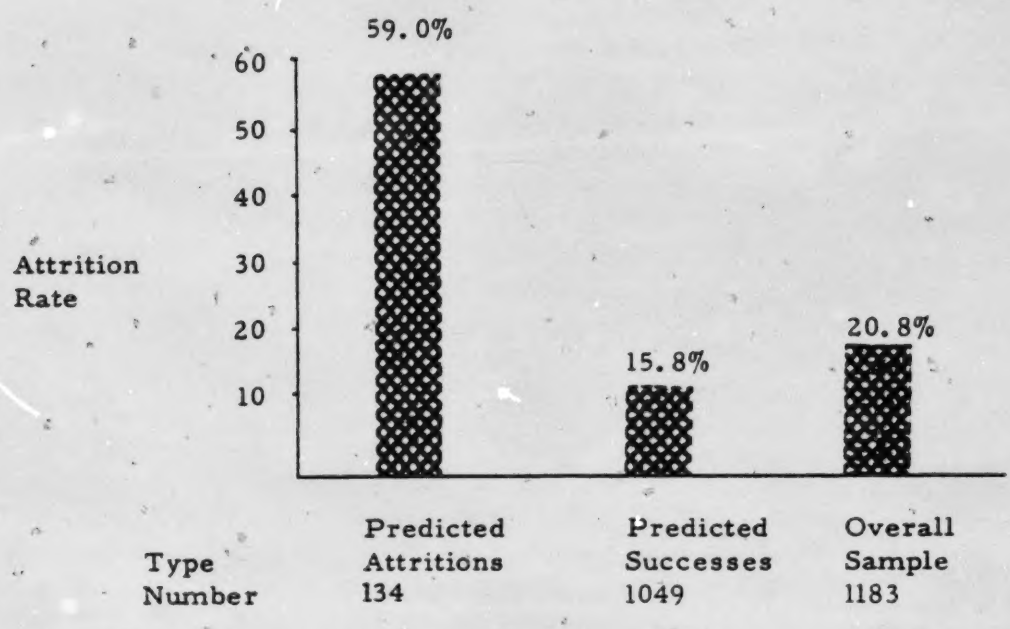
TABLE 2

PREDICTION RESULTS CLASS OF 1977

	Predicted Attritions	Predicted Successes	Total	Percent Correct
Actual Attritions	79	167	246	32.1%
Actual Successes	55	882	937	94.2%
Total	134	1049		
Percent Correct	59.0%	84.2%		

FIGURE 1

ATTRITION RATES CLASS OF 1977



IV. THE UNITED STATES AIR FORCE ACADEMY EMPIRICAL TEST

This section describes a follow-on test to further evaluate the conceptual approach and estimation procedure for possible application to other Air Force programs.

Background

Based on the results of the initial test described in the previous section the feasibility of the approach had been demonstrated. The empirical test described herein was designed to demonstrate that the methodology could, in fact, predict attrition a priori on a by-name basis. It was important to evaluate the procedure in a simulated operational environment which would require a two year lag in the prediction system. For these reasons the empirical test was conducted using the Class of 1977 to estimate the prediction equation and critical limit and using the Class of 1979 as the demonstration class.

Data

The empirical test utilized the same data and format collected for the Class of 1977 in the initial test. Identical data was collected on the Class of 1979 and a similar record constructed for each cadet. However there was one difference in the method of construction. Any cadet missing one or more of the principle variables was discarded from the sample in the initial test. Because the purpose of the empirical test was to simulate an operational environment in which all candidates would receive a prediction, any record missing a principle variable was given the mean value of that data element.[10/] This resulted in a 99.8 percent[11/] sample of the entering Class of 1979. (Table 3)

10/ Since estimation procedure converts independent variables to deviation form, this resulted in no weight being given to that data element in the individual's prediction of attrition.

11/ Three cadets had no data and were excluded from the test but this was not believed to significantly affect the results.

TABLE 3

SAMPLE SIZES FOR THE EMPIRICAL TEST

	1977	1979
Cadets Still Enrolled	937	1237*
Motivational Attrition	246	178
TOTAL IN SAMPLE	1183	1460**

* At completion of test
** Total in 1979--there were also 25 attritions for other reasons.

Test Methodology

A prediction system was estimated using the Class of 1977 which was then applied to the members of the Class of 1979 within three weeks after their arrival. The duration of the empirical test was approximately six months which allowed sufficient time to adequately assess the performance of the procedure. The test was terminated on 12 December 1975.

Results

The procedure was able to correctly classify 36.0 percent of the motivational attritions and 91.3 percent of the actual successes. (Table 4) Over 37 percent of the predicted attritions had separated by the end of their first semester. (Figure 2) Notably, thirteen additional predicted attritions separated shortly after their return from Christmas leave of which seven were motivational.

A recent Government Accounting Office (GAO) study concerning causes of Academy attrition[12] listed as one of its recommendations that:

> "The Secretaries of Commerce, Defense and transportation direct the academies to consider

[12] Comptroller General of the United States, Report to the Congress: Student Attrition at the Five Federal Service Academies, (Washington D.C.: Government Accounting Office, 5 Mar 1976).

methods to identify in the first days of summer, students with low commitment and to provide these students with counseling which might encourage them to stay."

The empirical test demonstrated that approximately one-third of these cadets can be identified within three weeks after their arrival with no change in existing Air Force Academy selection or testing procedures.

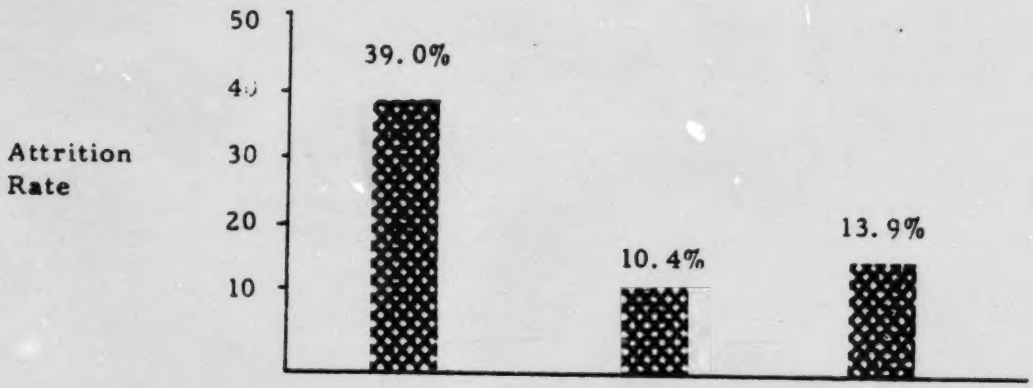
TABLE 4

PREDICTION RESULTS CLASS OF 1979
(INCLUDING ONLY MOTIVATIONAL ATTRITIONS)

	Predicted Attritions	Predicted Successes	Total	Percent Correct
Actual Attritions	64	114	178	36.0%
Actual Successes	110	1147	1257	91.3%
Total	174	1261		
Percent Correct	37.0%	91.0%		

FIGURE 2

ATTRITION RATES CLASS OF 1979



Type	Predicted Attritions	Predicted Successes	Overall Sample
Number	180	1280	1460
Attritions*	70	134	204

*Includes all attritions

V. CONCLUSIONS AND IMPLICATIONS FOR OTHER APPLICATIONS

Exploratory investigations at the United States Air Force Academy were designed to test an analytical framework which could be used to identify those candidates who are most likely to attrit from various Air Force training programs. The investigations are considered exploratory because the initial and empirical tests were designed only to demonstrate the feasibility of the approach. The two tests prove the validity of the conceptual approach and estimation procedure and represent an advance in the ability to model the phenomenon of attrition. Although the tests at the Air Force Academy only addressed motivational or voluntary attrition, academic attrition could be incorporated into the prediction system by expanding the procedure to accommodate a system of equations whereby the propensity of an individual to attrit is based on two determinants--motivation and ability.

The procedure described in this report could be used by any service academy to identify those cadets who exhibit a low commitment toward the academy in their first few weeks of training, and form the basis for providing these cadets with counseling aimed at enhancing their motivation. The procedure might eventually prove useful in the selection process as well.

Equally if not more important than the results obtained at the Academy are their implications for other programs which experience a high and costly attrition. The conceptual approach and estimate procedure can be applied directly to predicting voluntary attrition from a number of technical training schools. The conceptual approach and estimation procedure can also be applied to involuntary attritions, if the situation is such that there is no avenue for an individual to voluntarily leave the program. In these situations such as basic military training, a portion of the involuntary discharges will be of a voluntary nature and will result from an overt act or demonstration of adverse behavior on the part of an individual who lacks the motivation to complete the training.

Efforts are currently being directed into a number of these areas and preliminary evaluations indicate that results similar to those obtained at the Air Force Academy are probable.

APPENDICES

APPENDIX A

DIFFICULTIES WITH REGRESSION

The theoretical formulation of Classical Ordinary Least Squares (OLS) Regression requires that several assumptions be made with respect to the nature of the error term. These assumptions are stated concisely below:

$$\text{(i)} \quad E(\xi) = \emptyset$$

$$\text{(ii)} \quad E(\xi\xi') = \sigma^2 I$$

where ξ is a n x 1 vector of independent random variables and I is the Identity matrix.

In the case of a dichotomous regressand, defined to be 1 or $\emptyset$, the error term must assume a value of either:

$1 - X'\beta$ (Y observed equals 1)

or

$-X'\beta$ (Y observed equals $\emptyset$)

Thus in order for ξ to have an expectation of zero its distribution must be:

ξ	$f(\xi)$	
$(1-X'\beta)$	$X'\beta$	(Y observed equals 1)
$-X'\beta$	$(1-X'\beta)$	(Y observed equals $\emptyset$)

where $f(\xi)$ equals the normal p.d.f. evaluated at ξ which results in a variance of:

$$\begin{aligned} E(\xi\xi') &= -X'\beta^2 (1-X'\beta) + (1-X'\beta)^2 X'\beta \\ &= X'\beta(1-X'\beta) \\ &= E(Y)(1-E(Y)) \end{aligned}$$

Because the variance is a function of the expected value of Y, this implies that the variance varies systematically with the explanatory variables, X. Hence the assumption of homoskedasticity is untenable.

The Generalized Least Squares (GLS) model overcomes this difficulty by normalizing the variance to a constant by weighting the estimated Y by:

$$\frac{1}{E(Y)(1-E(Y))}$$

Nevertheless, there still remains (1) the problem of the Bernoulli nature of the error term with respect to hypothesis testing, (2) the assumption that the expectation of the error term equals Ø and (3) the E(Y) is unknown and has to be estimated. But the most serious deficiency springs from the misspecification of the functional form which does not prevent the estimates from varying outside of the unit interval and presumes a linear relationship between the explanatory variables and the true probability function. Moreover, Nerlove and Press illustrate that the slope of the estimated OLS regression line is sensitive to variations in the proportion of observed 1's and Ø's. Due to these and other conceptual and statistical difficulties it was appropriate to compare the performance of OLS to the results obtained with the MLE procedure described in Appendix D.

The comparison used the Classes of 1976 and 1977. A prediction equation and critical limit were estimated for the Class of 1976 using the linear OLS approach with the same specification contained in Appendix B. To provide a basis for comparison the critical limit was selected such that the error (false positive rate) of the OLS model equalled that of the MLE procedure. As expected the performance of OLS on the Class of 1976 compared very favorably to that of the MLE procedure and was reminiscent of the findings of Lockman, Jehn and Shughart. However, when the OLS prediction system was applied to the Class of 1977 the results deteriorated significantly. While MLE could identify 32.1 percent of the actual motivational resignations from the Class of 1977, OLS only identified 10.6 percent. Although, many variations of the comparison were carried out in no case did OLS outperform MLE.

The logit transformation[1/] could have been used by aggregating the data into mutually exclusive cells thus rendering an OLS solution

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[1/] Berkson, J., "Application of the Logistic Function to Bio-Assay," Journal of the American Statistical Association, 39, 1944.

possible. Such a model is specified below:

$$\ln\left[\frac{P}{1-P}\right] = X'\beta + e$$

. where P is an n x 1 vector of frequencies of occurrence.

Such an alternative, at best, represents an approximation to the procedure described in Appendix D. However this is not to say the Logistic function could not be substituted for the normal function in the MLE procedure. Such a substitution has its benefits manifested in the ease of calculations that result. But if there exists only one set of explanatory variables per observation a Least Squares solution is inappropriate. For further discussion concerning difficulties with OLS see Goldberger (1972), Nerlove and Press (1973), Tobin (1955) or Thiel (1971).

APPENDIX B

THE MEANS. STANDARD DEVIATIONS. COEFFICIENTS AND T-VALUES OF THE PRINCIPLE VARIABLES

$$Y = -.85\,X + .23\,X_1 + 1.05\,X_2 + .35\,X_3 + .48\,X_4 - .19\,X_5 + .17\,X_6 - .13\,X_7 + .09\,X_8 + .15\,X_9 - .09\,X_{10} + .08\,X_{11} + .06\,X_{12}$$

Critical Limit = .18
N = 1183
Chi Square = 114.20

X = intercept

X_1 = Recruited athlete (1 if Recruited, Ø Not)

X_2 = Interest in Military Activites (SVIB) (1 if ≤ 50, Ø otherwise)

X_3 = Interest in Mathematics (SVIB) (1 if ≤ 40, Ø otherwise)

X_4 = Interest in Science (SVIB) (1 if ≤ 35, Ø otherwise)

X_5 = Varsity swimmer in high school (1 if was, Ø otherwise)

X_6 = Class officer in high school (1 if was, Ø otherwise)

X_7 = Received outstanding student award in high school (1 if was, Ø otherwise)

X_8 = Valedictorian or Salutatorian in high school (1 if was, Ø otherwise)

X_9 = Junior or college AFROTC (1 if was, Ø otherwise)

X_{10} = Received scholarship offer from another college (Ø if was, 1 otherwise)

X_{11} = Prior Academic Record

X_{12} = College Entrance Exam (verbal)

	MEAN	S.D.	T-VALUE
X_1	.14	.35	1.65
X_2	.15	.36	2.54
X_3	.10	.30	2.06
X_4	.07	.26	2.17
X_5	.08	.28	1.66
X_6	.11	.32	1.65
X_7	.35	.48	1.68
X_8	.11	.32	1.62
X_9	.03	.16	1.75
X_{10}	.51	.50	1.62
X_{11}	582.86	91.83	1.55
X_{12}	577.79	66.59	1.54

APPENDIX C
FILE LAYOUT OF USAFA DATA

COLUMNS	DESCRIPTION	
1-2	Next to Last Two Digits of CCN	
3	Good=0, Bad=1	
4-6	PAR	
7-9	VRB	
10-12	Eng Ach	
13-16	Non-Scientific Index	
17-19	Math Apt.	
20-22	Math Achievement	
23-26	Sci Index	
27-30	Academic Composite	
31-33	PAE	
34-36	Athletic Index	
37-40	Non Athletic Index	
40-43	Leadership Composite	
44-47	Weighted Composite	
48-50	Panel Rating	
51	Math Achievement Level	1=Advanced; Ø=not advanced
52	Medical Status	1=Pilot; 2=NAV; 3=Non-Rated
53	Prep School	1=Yes; Ø=No
54	Prior College	1=Yes; Ø=No
55	Recruited Athlete	1=Yes; Ø=No
75		
76		
77		
82		
83		
84		
85		
90		
91		
92		
93		
94		
110		
114-116	Cadet GPA	
117-120	Blank	
121-122	Strong Variable	#1 Public Speaking
123-124	" "	#2 Law/Politics
125-126		#3 Business Management
127-128		#4 Sales
129-130		#5 Merchandizing
131-132		#6 Office Practice
133-134		#7 Military Activities
135-136		#8 Technical Supervision
137-138		#9 Mathematics
139-140		#10 Science
141-142		#11 Mechanical

143-144	Strong Variable
145-146	" "
147-148	
149-150	
151-152	
153-154	
155-156	
157-158	
159-160	
161-162	
163-164	
165-166	
167-168	
169-170	
171-172	
173-174	
175-176	
177-178	
179-180	
181-182	
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203-204	
205-206	
207-208	
209-210	
211-212	
213-214	
215-216	
217-218	
219-220	
221-222	
223-224	
225-226	
227-228	
229-230	
231-232	
233-234	
235-236	
237-238	
239-240	
241-242	

#12 Nature
#13 Agriculture
#14 Adventure
#15 Recreational Ldr.
#16 Medical Service
#17 Social Service
#18 Religious Activites
#19 Teaching
#20 Music
#21 Art
#22 Writing
#23 Dentist
#24 Osteopath
#25 Vetinarian
#26 Physician
#27 Psychiarist
#28 Psychologist
#29 Biologist
#30 Architect
#31 Math Teacher
#32 Physicist
#33 Chemist
#34 Engineer
#35 Production manager
#36 Army Officer
#37 Air Force Officer
#38 Carpenter
#39 Forest Serviceman
#40 Farmer
#41 Math/Science Teacher
#42 Printer
#43 Policeman
#44 Personnel Director
#45 Public Administrator
#46 Rehabilitation Worker
#47 YMCA Staff Member
#48 Social Worker
#49 Social Science Teacher
#50 School Superintendent
#51 Minister
#52 Librarian
#53 Artist
#54 Musician
#55 Music Teacher
#56 C.P.A. (owner)
#57 Senior C.P.A.
#58 Accountant
#59 Office Worker
#60 Purchasing Agent
#61 Banker

243-244	Strong Variable	#62 Pharmacist
245-246	" "	#63 Funeral Director
247-248		#64 Sales Manager
249-250		#65 Real Estate Sales
251-252		#66 Life Insurance
253-254		#67 Adver. mgr.
255-256		#68 Lawyer
257-258		#69 Author/Journalist
259-260		#70 President (mfg)
261-262		#71 Credit manager
263-264		#72 Chamber of Commerce
256-266		#73 Physical Therapist
267-268		#74 Programmer
269-270		#75 Business Education
271-272		#76 Community/Rec. Adm.
	(Non-Occupational Skills)	
273-274		#77 Academic
275-276		#78 Age Related
277-278		#79 Diversity of Interest
279-280		#80 Masculinity/Femininity
281-282		#81 Managerial Skill
283-284		#82 Occ-intro/extro
285-286		#83 Occ level
287-288		#84 Spec. level
289-290	Squadron Number July 72	
291-292	Squadron Number Fall 73	
293	Football	0=No,1=Yes
294	Basketball	0=No, 1=Yes
295	Baseball	0=No, 1=Yes
296	Track	0=No, 1=Yes
297	Hockey	0=No, 1=Yes
298	Golf	0=No, 1=Yes
299	Tennis	0=No, 1=Yes
300	Swimming	0=No, 1=Yes
301	Wrestling	0=No, 1=Yes
302	Other Sport	0=No, 1=Yes
303	Yearbook or Newspaper Staff	0=No, 1=Yes
304	Band, Orchestra, Debate Team, Dramatic Production, Chorus, Speech Contests, or Cheerleader	0=No, 1=Yes
305	Science Club President	0=No, 1=Yes
306	Language Club President	0=No, 1=Yes
307	Hobby Club President	0=No, 1=Yes
308	Service Club President	0=No, 1=Yes
309	Career Interest Club President	0=No, 1=Yes
310	Honorary Organization President	0=No, 1=Yes

311	Student Government Officer	0=No, 1=Yes
312	Athletic Club	0=No, 1=Yes
313	President of Class	0=No, 1=Yes
314	VP, Secy,Treas of Class	0=No, 1=Yes
315	Delegate to Boys State	0=No, 1=Yes
316	Citizenship Award	0=No, 1=Yes
317	Outstanding Student Award	0=No, 1=Yes
318	Valedictorian or Salutatorian	0=No, 1=Yes
319-32-	Card No.	
321-322	Blank	
323-324	First Significant Influence on Academy Application	
	1- parents or relatives	
	2- high school coach	
	3- Boy Scouts	
	4- high school counselor or teachers	
	5- Civil Air Patrol	
	6- Congressman	
	7- Air Force Recruting Service	
	8- Principal	
	9- Self Generated interest	
	10- Military Officers	
	11- Academy liasion officers	
	12- Other AFA cadets	
	13- Other	
325-326	Second factor	
327-328	Third factor	
329-330	Age Interested	
331	Would guidance about AFA help in Jr. high school	
	0-Yes, 1-No, 2- Don't Know	
332	How many times seek appointment?	
	0-1,1-2,2-3,3-4,4-5	
333	Comment about LO.	
	0-No Knowledge	
	1-Have heard of him but never contacted	
	2-Was counseled prior to application but not after	
	3-Counseled before and after application	
	4-Counseled only after	
334	Was he helpful?	
	0-Extremely helpful	
	1-Sometimes unreliable	
2	2-Beneficial	
	3-Not Necessary	
	4-No contact	
335	Did you have contact with a cadet prior to arriving?	
	0=Yes	
	1=No	
336	Did you know about BCT?	
	0=Yes	
	1=No	

337	Who Advised you of BCT? 0= Cadet 1= Liasion offices 2= Brother
338	Did you apply to another academy? 0=Yes 1=No
339	Was father career military? 0=Yes 1=No
340-341	Highest grade 1= General 2= Colonel 3= Lt Colonel 4= Major 5= Captain 6= 1st Lieutenant 7= 2nd Lieutenant 8= Warrant officer 9= E-9 10= E-8 11= E-7 12= E-6 13= Below E-6
342	Service 0= Army 1= Air Force 2= Marines 3= Navy 4= Coast Guard
343	Is father serving on Active Duty? 0= Yes 1= No
344-345	Grade Same as 380-381
346	Service 0= Army 1= Air Force 2= Marines 3= Navy 4= Coast Guard
347	Was father Academy Grade? 0= West Point 1= Anapolis 2= Coast Guard 3= Merchant Marine 9= blank
348	Father POW or MIA 0= POW 1= MIA 9= No

349 Had a brother in the Academy?
0= No
1= Yes

350 Member of CAP
0= No
1= Yes

351 Member of Jr. ROTC
0= Army, 1= Air Force, 2= Navy, 9= No

352 ROTC in College?
0= Army, 1= Air Force, 2= Navy, 9= No

353 USAF Prep School?
0= Yes
1= No

354 Were you in National Merit?
0= Yes, but did not qualify
1= letter of Commendation
2= Semifinalist
3= Finalist
9= No

355 Scholarship to another institutional
0= Yes
1= No

356 What kind of Scholarship?
0= Academic
1= Athletic
2= Both
3= Other

357 Were you a multiple person?
0= Twin
1= Triplet

358 Race
0= Indian
1= Black
2= Asian American
3= Spanish American
9= White

APPENDIX D

MOTIVATIONAL ATTRITION PREDICTION (MAP) MODEL[1/]

Introduction

Motivational (voluntary) attrition from Air Force training programs can be considered simply as a change in career goals on the part of the individuals involved. Changes in career goals can be viewed through the classical Marshallian framework--"The attractiveness of a trade depends not on its money earnings, but its net advantages."[2/] Initially the individual surveys the alternatives available to him and weighs the advantages and disadvantages of each. In his assessment of the respective alternatives he considers not only monetary factors, but also non-monetary factors such as prestige, locations, and perhaps security. Naturally he selects the one with the highest net advantage.

For purposes of illustration consider the recurring decision facing a cadet enrolled at the Air Force Academy. Assume he makes an implicit dollar valuation incorporating all of the advantages and disadvantages of his current career choice and a similar valuation for an alternative choice, given his knowledge of each. So long as his subjective dollar valuation of his current career choice (call this his Academy utility) is greater than the subjective dollar valuation of the alternative career being considered (call this the Alternative utility) he remains at the Academy. The decision is made in terms of the relative difference between the two utilities. As long as the net difference is positive he will not attrit; if it is zero he is indifferent and if it is negative he will voluntarily leave the Academy or perform in such a way that will achieve this end.

The Model

Let Y be a dichotomous random variable defined to be 1 if an event E occurs; and $\emptyset$ otherwise. Let X be a 1 x m vector of m explanatory variables of Y which may be dichotomous, polytomous, or continuous.[3/] Let β be a m x 1 vector of coefficients such that $X'\beta$ specifies a linear function of X. Finally, let ξ denote a n x 1 vector of random disturbances distributed $N(\emptyset, 1)$. By hypothesis, Y_i is related to $X'\beta_i$ $(i = 1, \ldots, n)$, such that:

1/ The estimation procedure described in this Appendix has been programmed in Fortran IV (ASCII) on a Burroughs 6700 computer.

2/ Marshall, Alfred, Principles of Economics, 8th ed., (London: MacMillian and Company, 1961), p. 557.

3/ To satisfy the assumption of normality, the value of the dependent variable should be able to assume 30 different values.

Observed

$Y_i = 1$: when $X'\beta_i + \xi_i \succ U_i$ (event occurs)

$Y_i = \emptyset$: when $X'\beta_i + \xi_i \preceq U_i$ (event does not occur)

where U_i represents a n x 1 vector of utilities that the individuals receive from the event not occurring and is $\sim N(\emptyset, \sigma^2)$.

Conceptually, when an individual is faced with two alternative choices he will assign a utility to each. Since we assume that the individual will act rationally and seek to maximize his total utility, he will be expected to select the alternative to which he assigned the highest utility. Although, from the individual's point of view, the choice is purely deterministic, from the observer point of view, the choice has a systematic component, $X'\beta_i$, and a random component, $U_i - \xi_i$. If we attempt to apply utility maximization to the known component we will predict a fraction of the cases correctly.

Let P_i represent the probability to an event E occurring such that:

$$P_i = \text{prob}\,(X'\beta_i + \xi > U_i) = \text{prob}\,(X'\beta_i > U_i - \xi_i)$$

which can be further expressed by (1.1).

$$(1.1) \quad P_i = \int_{-\infty}^{\infty}\int_{-\infty}^{U_i - X'\beta_i} f(\xi_i, U_i)\, d\xi_i\, dU_i$$

where $f(\xi_i, U_i)$ is the joint density function of ξ_i and U_i.

Since we have a systematic component, $X'\beta_i$, and a random component, $U_i - \xi_i$, we can reduce (1.1) to a more manageable level by making the substitution $\xi'_i = U_i - \xi_i$. The new random component, ξ'_i, is assumed to be distributed $N(\emptyset, \sigma'^2)$. 4/

Thus equation (1.1) reduces to:

$$(1.2) \quad P_i = \int_{-\infty}^{X'\beta_i} f(\xi')\, d\xi'$$

4/ The mean of ξ' is denoted by μ' where:

$$\mu' = E(U) + E(\xi)$$

and the variance of ξ' is denoted by σ'^2 where:

$$\sigma'^2 = \text{var}(U) + \text{var}(\xi)$$

Since we have chosen the normal transformation because of its basis in nature, we then elect to normalize the integral in (1.2) by letting:

$$z = \frac{\xi' - u'}{\sigma'} = \frac{\xi' - \emptyset}{\sigma'}$$

$$dz = \frac{1}{\sigma'}\, d\xi'$$

Finally, if we equate the occurrence of event E to an individual's failure from a training program, P_i represents his probability of failure as a function of his unique set of personal characteristics weighted by a vector of coefficients, β. Thus:

$$(1.4) \quad P_i = F\left(\frac{X'\beta}{\sigma'}_i\right)$$

Before solving for the respective coefficients we make the following substitutions for notational convenience:

(1) Let $J_i = \frac{X'\beta}{\sigma'}_i \qquad (i = 1, \ldots, n)$

(2) Let $\alpha_{m+1} = \frac{1}{\sigma'}$

(3) Let $\alpha_k = \frac{\beta}{\sigma'} \qquad (k = \emptyset, \ldots, m)$

(4) Let $I_i = X'\beta_i + \xi_i$

The Maximum Likelihood Solution

Let S represent an ordered sample of T observations, where the first r observations equal zero and the remaining T-r observations equal one. Without loss of generality, the likelihood of the sample is given by:

$$(1.5) \quad L = \prod_{i=1}^{r} \left[1 - F(J_i)\right] \times \prod_{i=r+1}^{T} F(J_i)$$

In order to maximize L in terms of α_k, the logarithmic likelihood must be derived and is given in (1.6):

$$(1.6) \quad \ln L = \sum_{i=1}^{r} \left[1 - F(J_i)\right] + \sum_{i=r+1}^{T} F(J_i)$$

(1.7) where: $F(u) = \int_{-\infty}^{\mu} \frac{1}{\sqrt{2\pi\sigma'}} e^{-\frac{1}{2}(\frac{\mu}{\sigma'})^2} d\mu$

and

(1.8) $f(u) = \frac{1}{\sqrt{2\pi\sigma'}} e^{-\frac{1}{2}(\frac{\mu}{\sigma'})^2}$

Let X_o be exactly 1 for all i. Then setting the partial derivatives of lnL with respect to α_k equal to $\emptyset$ yields the following system of m + 1 equations:

$$(1.9) \quad \frac{\partial \ln L}{\partial \alpha_k} = \sum_{i=1}^{r} \frac{-f(J_i)}{\left[1-F(J_i)\right]} X_{ki} + \sum_{i=r+1}^{T} \frac{f(J_i)}{F(J_i)} X_{ki} = \emptyset$$

$$\vdots$$

$$\frac{\partial \ln L}{\partial \alpha_{m+1}} = \sum_{i=1}^{r} \frac{-f(J_i)}{\left[1-F(J_i)\right]} X'\beta_i + \sum_{i=r+1}^{T} \frac{f(J_i)}{F(J_i)} X'\beta_i = \emptyset$$

These equations are, of course, non-linear but can be solved simultaneously by the Newton-Raphson method.

Tests of Hypothesis[5/]

Tests of hypothesis reagarding the significance of one or more of the predictor variables may be tested once the parameter variances and convariances are specified. The respective square roots of the diagonal elements of the negative inverse of the matrix of second derivatives evaluated at the point of maximum Likelihood yields large sample estimates of the coefficient standard errors. Once computed, standard tests of hypotheses for one or more of the predictor variables can be easily accomplished.

$$(1.10) \quad \frac{\partial^2 \ln L}{\partial \alpha_\kappa \alpha_\tau} = \sum_{i=1}^{r} -X_k X_t \left[\frac{f(J_i)^2}{1-F(J_i)^2} - \frac{f(J_i)J_i}{1-F(J_i)}\right] + \cdots$$

$$+ \sum_{i=r+1}^{T} X_k X_t \left[\frac{f(J_i)J_i}{F(J_i)} + \frac{f(J_i)^2}{F(J_i)^2}\right] \quad (t, k = \emptyset, 1, \ldots, m)$$

5/ Tests of hypotheses may also be accomplished for any set $(\alpha_\kappa, \ldots, \alpha_m)$ by the Likelihood Ratio Method.

Prediction

Since we are primarily interested in which alternative the i'th individual will choose, rather than characteristic statistics of the group, we must develop a prediction mechanism whereby we may infer within some fudicial limit which alternative he will choose.

In estimating β we have assumed X to be fixed. We may relax this constraint as long as we can assume that X is uncorrelated with β, ξ, and U; such that we consider the conditional probabilities of our estimators given X. For example: $E(\hat{b}_1/X_1) = \beta_1$, where X_1 is an nxm vector of given X's, and σ_i (for i = 1, . . . , m) represents the standard deviation of the respective explanatory variables. Moreover, our estimators still possess the desired properties of efficiency and consistency. 6/

By relaxing the assumption that X is fixed and realizing that we, the observers, have no control over what value X assumes, we may say that the utilities among different individuals for the alternatives choices are distributed as independent bivariate normal random variables, such that:

$$(1.11) \qquad \text{prob}\,(I_i \succ U_i) = \int_{-\infty}^{\infty}\int_{-\infty}^{U_i} f(I_i, U_i)\,dI_i\,dU_i$$

Using the convolution formula and letting $w_i = I_i - U_i$ we find that the marginal density is given by:

$$(1.12) \qquad f(w_i) = \int_{-\infty}^{\infty} f_1(w_i + U_i).\, f_2 U_i\, dU_i \quad \text{where } f_1 = f_2 = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{\mu}{\sigma}\right)^2}$$

6/ A simple proof is based on Chebyshev's inequality, which states that for any random variable Z with a finite mean ν and variance σ^2, the probability of a deviation equal to K times the standard deviation or more is at most equal to $1/K^2$:

$$P\left\{|Z - \nu| \geq K\sigma\right\} \leq 1/K^2 \quad \text{for any } K \succ \emptyset$$

In the case of $\hat{b}$, and its mean, β the variance is σ^2/N, so that the inequality in brackets becomes $|\hat{b} - \beta| \geq K\sigma/\sqrt{N}$. By specifying $K = e\sqrt{N/\sigma^2}$ we thus obtain:

$$P\left\{|\hat{b} - \beta| \geq e\right\} \leq \frac{\sigma^2}{Ne^2} \quad \text{for any } e \succ \emptyset$$

Since σ^2/Ne^2 goes to zero as $N \to \infty$ we may conclude $\hat{b}$ is a consistent estimator of β.

Thus, the marginal density of $w_i = I_i - U_i$ is:

$$(1.13) \qquad g(w_i) = \frac{1}{\sigma^* \sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{W_i - \mu^*}{\sigma^*}\right)^2}$$

where $E(\mu^*) = \int_{-\infty}^{\infty} I_i f(I_i) - \int_{-\infty}^{\infty} U_i f(U_i) = b_o$ [7/]

$\sigma' = \sqrt{\text{var}(U) + \text{var}(\xi)}$ [8/]

and $\sigma^* = \sigma' + \sqrt{\sum_{i=1}^{m} \sigma_i^2}$ σ_i = std dev. of X_i

We now proceed to interpret the marginal density of w.

Considering that w represents the difference between the respective utilities, when the sum equals $\emptyset$ the individual is said to be indifferent between the two alternative choices. Thus:

$g(\emptyset)$

is the mean point of indifference for all individuals and is estimated by $f(b_o)$, where $f(.)$ is the $N(\emptyset, 1)$ probability density function (p. d. f.).

However, before we make our predictions we must take into account the uncertainty in (1) the mean point of indifference, (2) the estimators, and (3) the random disturbances.

First, we construct an upper confidence bound on $\hat{b}_o$ such that:

$$(1.14) \qquad \hat{b}_o^* = \hat{b}_o + z_\alpha \sqrt{\text{var}(\hat{b}_o)}$$

Second, we construct a lower confidence bound on $\hat{I}_i$ given X_i such that:

$$(1.15) \qquad \hat{I}_i^* = \hat{I}_i - z_\alpha \sigma' \sqrt{\sum_{j=0}^{m} \text{var}(\hat{b}_j) X_j + 1}$$

[7/] $\hat{b}_o$ equals the intercept which X is in deviation form.
[8/] for large samples only

Possessing all the components we now compare $F(\hat{I}_1^*)$ to $F(\hat{b}_o^*)$ and predict under the following regime:

If $F(\hat{I}_1^*) \succ F(\hat{b}_o^*)$ the event is predicted to occur, i. e. $\hat{Y} = 1$

If $F(\hat{I}_1^*) \leq F(\hat{b}_o^*)$ the event is predicted not to occur, i. e. $\hat{Y} = 0$

where $F(.)$ is the $N(\emptyset, 1)$ Cumulative Density Function (c. d. f.)

The results should be interpreted as follows: At the Air Force Academy, a candidate with a given set of personal attributes will be predicted to attrit prior to graduation if his estimated utility for his set of alternatives is greater than the estimated mean point of indifference. The confidence which can be placed in each prediction of attrition is an option which can be varied along with the percent of actual attritions identified and the percent false positive (i. e., those predicted attritions who actually succeed). Conceivably, these three parameters could be put into an optimizing framework to yield the best result given a user's preferences.

APPENDIX E

UNITED STATES AIR FORCE ACADEMY DISCHARGE CODES
ATTRITION DESCRIPTION

DESCRIPTION

Medical-Discharge
Conduct-Discharge
Academic-Discharge
Aptitude-Discharge
Aptitude & Conduct-Discharge
Aptitude & Academic-Discharge
Conduct & Academic-Discharge
Failure in Summer Training-Discharge
Failure in Physical Education-Discharge
Honor-Discharge
Honor-Lying
Honor-Stealing
Honor-Cheating
Honor-Toleration
Honor-Lying & Stealing
Honor-Lying & Cheating
Honor-Lying & Toleration
Honor-Lying, Stealing & Cheating
Honor-Lying, Cheating & Toleration
Honor-Lying, Stealing, Cheating & Toleration
Honor-Stealing & Cheating
Honor-Stealing & Toleration
Honor-Stealing, Cheating & Toleration
Honor-Cheating & Toleration
Honor-Used Honor Code as a means of departing
Aptitude-Conduct-Academic

DESCRIPTION

ACADEMIC RESIGNATIONS

Insufficient Choice of Courses
Dislike Instructional Methods
Pressure of Academic Systems
Reserved
Inability to Cope with Academics-Deficient

ENVIRONMENTAL ADJUSTMENT-RESIGNATIONS

Unwilling or Unable to Make a Satisfactory Group Adjustment
Too Much Regimentation & Lack of Personal Freedom
Too Much Competition
Disappointed in Caliber of Cadets, Both Peers and Upperclass
Reserved

CAREER GOALS-RESIGNATIONS

Lack of Desire or Motivation
Insufficient Desire to Complete the Academy Program
Always Desired Another Career
Changed Career Interest After Entering
Change in Physical Condition Not Requiring Separation

DESCRIPTION

HONOR RESIGNATIONS

Honor Resignation
Lying
Stealing
Cheating
Toleration
Lying & Stealing
Lying & Cheating
Lying & Toleration
Lying, Stealing & Cheating
Lying, Stealing, Cheating & Toleration
Lying, Cheating & Toleration
Stealing & Cheating
Stealing & Toleration
Stealing, Cheating & Toleration
Cheating & Toleration
Used Honor Code as a Means of Departing

PERSONAL RESIGNATION

Personal Resignation
Marriage (Married)
To be Married
Lack of Confidence (Immaturity)
Hardship
Good of Service
Inability to Cope with Military Training Program
Unable or Unwilling to Accept All Tenets of the Honor Code. (Do not count as Honors)

OTHER RESIGNATIONS

Other-Unclassified
Resignation in Lieu of Board Action for Lack of Military Aptitude

DESCRIPTION

OTHER RESIGNATIONS (Cont)

Conscientious Objector
Anti-Military Feelings
Parental Pressure
Deceased

STRENGTH ADJUSTMENT CODES

Departed-Pending Turnback
Turnback
Turnforward
Departed Cadet Returned and Turned Back
Departed Cadet Returned and Remains with Class at Time of Departure
Re-entry of Cadet who Previously Resigned or was Discharged
Foreign Exchange Student
Graduated & Commissioned USAF
Graduated but Deceased at Time of Graduation
Graduated but not Commissioned
Graduated & Commissioned Other Service

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